

Realization of an Op-Amp Based PID Controller

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Abstract - In this paper I present an easy-to-fabricate PID controller composed of two operational amplifiers and a couple of resistors and capacitors. Rural communities that cannot afford commercially available PID controllers may want to explore this option for their control and production needs. Based upon easily available components, such a controller could be easily utilized in homemade appliances where cost and complexity might be issues to contend with. It may also be applied in scenarios where PLC and ladder logic might be considered overkills or overly expensive. The main challenge to using this PID controller remains the need for the plant parameters to be know in the frequency domain. Also presented is a MATLAB utility which assists in tuning the PID controller. It may be wise to keep the Ziegler-Nichols method as a rule of thumb (where the integral and derivative gains are initially set to zero) when using this MATLAB utility.

Index Terms: PID, op-amp, step-response, steady-state error

1.0 INTRODUCTION

In this paper I engage a low-cost op-amp based PID controller and a MATLAB utility that helps in the determination of the op-amp circuit values. Based upon easily available components, such a controller could be easily utilized in homemade appliances where cost and complexity might be issues to contend with. It may also be applied in scenarios where PLC and ladder logic might be considered overkills or overly expensive. The main challenge to using this PID controller remains the need for the plant parameters to be know in the frequency domain.

The rest of the paper is outlined as follows. First I introduce op-amp circuit and demonstrate that it's transfer function mimics the basic PID equation. Thereafter, I introduce a

MATLAB utility which enables trial-and-error determination of PID controller parameters, together with an examination of step-function response. Thirdly, I apply the estimates of the PID parameters in the determination of resistor and capacitor values in the op-amp circuit.

2.0 CIRCUIT AND PID EMULATION

It can be demonstrated that the transfer function of the RC op-amp circuit in figure-1 has the same form as the classical PID equation:

$$G_c(s) = sK_d + K_p + \frac{K_i}{s} \dots\dots(1)$$

Where $G(s)$ is the controller transfer function, K_p is the proportional gain, K_i is the integral gain and K_d is the derivative gain. Upon analysis, the transfer function of the op-amp circuit in figure-1 is given by:

$$G_c(s) = - \left[sC_1R_2 + \frac{C_1R_1 + C_2R_2}{C_2R_1} + \frac{1}{sC_2R_1} \right] \dots\dots(2)$$

The inverted output of equation (2) is easily corrected by the addition of an inverting unity-gain buffer (shown in figure-2) to the output of the op-amp circuit of figure-1. Now, comparing equations (1) and (2) on a termwise basis enables the evaluation of the classical PID parameters:

$$K_d = C_1R_2 \dots\dots(3)$$

$$K_p = \frac{C_1R_1 + C_2R_2}{C_2R_1} \dots\dots(4)$$

$$K_i = \frac{1}{C_2R_1} \dots\dots(5)$$

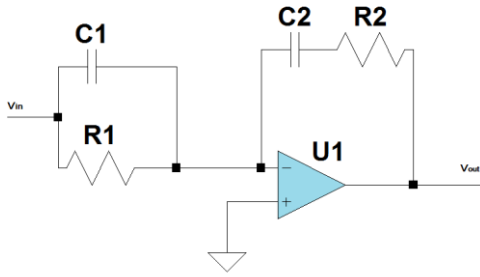


Figure-1: op-amp circuit for PID control

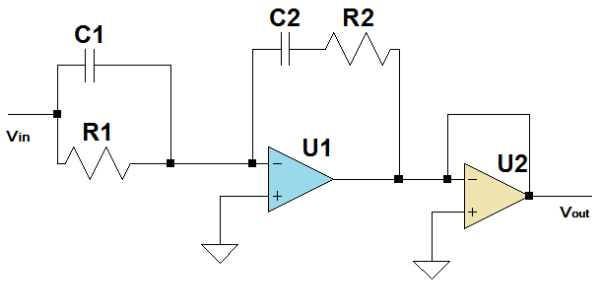


Figure-2: op-amp circuit, with inverting buffer to correct for initial inversion

3.0 PID TUNING USING MATLAB UTILITY

In this section, as already mentioned, it is assumed that the plant frequency domain model has already been identified. Classic texts[1][2] for the classification exercise may be consulted by the reader. As a working example, I will take the incubator frequency domain model as determined by Okpagu and Nwosu[3] both of whom came up with the following model for an egg incubator:

$$G_p(s) = \frac{1}{120s + 1} \quad \dots (6)$$

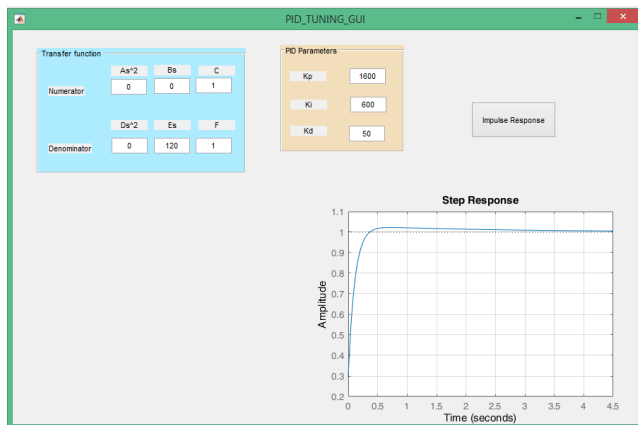


Figure-3: MATLAB utility for PID tuning and impulse-response simulation

Keying in the plant parameters into the blue colored box of the MATLAB utility is straightforward. The numerator section holds the coefficients of the numerator polynomial while the denominator section holds the coefficients of the denominator polynomial. I have limited both polynomials to the second order, but the m-file may be easily modified for higher order polynomials. Having keyed in the plant model, the user has to carefully key in educated estimates of the PID control parameters. In the case of the incubator model in equation (6), I have tried several PID parameters and I am satisfied with the impulse-response graph displayed in the lower right of figure-3. In the next section, I will utilize the PID control parameters to determine resistor and capacitor values.

It may be wise to keep the Ziegler-Nichols[4] method as a rule of thumb, where the integral and derivative gains are initially set to zero. The proportional gain, K_p is then increased (from zero) until it reaches an *ultimate* gain K_u , at which the output of the control loop has stable and consistent oscillations. K_u and the oscillation period T_u are then used to set the K_p , K_i , and K_d gains depending on the type of controller used.

4.0 RC CIRCUIT DETERMINATION

From figure-3, we can now make use of the following equations:

$$K_d = 50 = C_1 R_2 \quad \dots (7)$$

$$K_p = 1600 = \frac{C_1 R_1 + C_2 R_2}{C_2 R_1} \quad \dots (8)$$

$$K_i = 600 = \frac{1}{C_2 R_1} \quad \dots (9)$$

Assigning values to the resistors (say $1K\Omega$ to R_1 and $10K\Omega$ to R_2), solving for C_1 and C_2 becomes a trivial affair using only equations (7) and (8):

$$C_1 = 4997.55 \mu F \quad \dots (10)$$

$$C_2 = 1.667 \mu F \quad \dots (11)$$

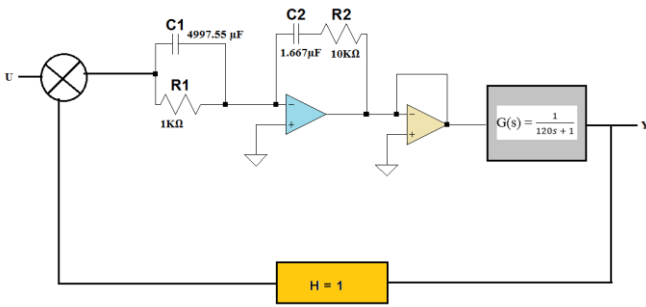


Figure-4: block diagram of feedback controlled incubator model by Okpagu et al

5.0 CIRCUIT REUSABILITY

Having selected the proper resistor and capacitor component values to match the plant model under consideration, the question as to how such a hard-coded PID controller will remain useful if and when the plant parameters alter due to any number of factors. From the point of view of design reusability, the hard-codedness of this PID controller may come across as a challenge. I would contend that this hard-codedness might remain a price to be paid for this low-cost PID controller solution.

The user has the responsibility of ensuring that the plant parameters do not vary widely; and when they do s/he must take steps to adjust the op-amp circuit values appropriately. It may be wise to make use of electronic trimmers so that small adjustments can be made to the resistance and capacitance values. However, care must be take to monitor the values of K_d , K_p and K_i in order to ensure overall system stability.

- > Re-use MATLAB utility
- > Re-tune the PID controller manually
- > A new one has to be put together for any new plant.

6.0 STEADY-STATE ERROR

Given that the step function was used to excite the feedback control system in the MATLAB simulation, it is appropriate to pay closer attention to the steady-state error of the entire system. The steady-state error is the control signal error as the system transient response decays, leaving only the system's continuous response. For the system in figure-4, the error value given by the expression:

$$E(s) = \frac{U(s)}{1 + G_c(s)G_p(s)} \dots (12)$$

Where $G_c(s)$ and $G_p(s)$ are given by equations (2) and (6) respectively while $U(s)$ is given by the Laplace transform of the step-function:

$$U(s) = \frac{1}{s} \dots (13)$$

In order to determine the steady-state error, the final-value theorem will be utilized:

$$\lim_{t \rightarrow \infty} e(t) = \lim_{s \rightarrow 0} sE(s) \dots (14)$$

So that:

$$e(\infty) = \lim_{s \rightarrow 0} s \left(\frac{1}{s} \left\{ \frac{1}{1 + G_c(s)G_p(s)} \right\} \right) \dots (15)$$

From equation (15), becomes clear that the $e(\infty)$ comes closer to zero whenever the term $G_c(s)G_p(s)$ assumes a large enough value as s tends to zero. Now, equations (2) and (6) clearly reveal that:

$$\lim_{s \rightarrow 0} G_p(s) = 1 \dots (16)$$

$$\lim_{s \rightarrow 0} G_c(s) = \lim_{s \rightarrow 0} \left[sC_1R_2 + \frac{C_1R_1 + C_2R_2}{C_2R_1} + \frac{1}{sC_2R_1} \right] \dots (17)$$

As s tends to zero, the value of expression (17) tends to infinity, also increasing the magnitude of the product $G_c(s)G_p(s)$. Under these circumstances, equation (17) becomes immune to the vagaries of capacitor and resistor tolerances. In other words, regardless the non-ideal behavior of the capacitors and resistors in the op-amp circuit, the steady-state error of the control system will tend toward zero as time tends to infinity. Therefore, at least in principle, there is no need to worry about compensating circuitry for such tolerances, or embracing using another design scheme, or bothering much about choosing choose resistors and capacitors narrow tolerance ranges.

7.0 OTHER PRACTICAL CONSIDERATIONS

The circuitry presented in this paper is suitable for single-input-single-output (SISO) control systems. In addition, a designer might want to couple this controller to the output of a low-cost programmable-logic-controller (PLC), one which might not ship with a PID control block in its ladder logic implementation.

In practice, the designer of this op-amp based PID circuit must exercise prudence in matching the op-amp

specifications to the industrial application under consideration; whether to utilize a low-power or high-power op-amp. Data sheets for real world op-amps will come handy at this juncture[5] and the designer must make a decision regarding single-power-supply[6] or dual-power-supply op-amps.

As usual, in terms of impedance considerations, the op-amp PID must possess a high input impedance and low output impedance. Furthermore, adjustments must be made to render the op-amp output compatible with the signal input expected by the plant actuator. It is equally assumed that the response curves of the feedback sensor are well known. Lastly, provisions must be made for cooling the circuitry and rendering it physically rugged, given the stringent demands of industrial control.

8.0 CONCLUSION

In this paper, I have been able to demonstrate feasibility of a simple but effective op-amp based PID controller. This circuit would come handy in locations where the acquisition installation of a commercial PID controller would be a challenge. This circuitry makes it possible for low income rural farming communities to play around with tank water level control, incubators, water-pump speed control and so on. The MATLAB utility is available for download the the url:<https://www.mathworks.com/matlabcentral/fileexchange/67194-utility-for-pid-tuning>.

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